

Appendix I: Further details of mathematical procedures

Section 1 – Number

Reading and writing numbers

Pupils must be encouraged to write numbers simply and clearly.

Most pupils are able to read, write and say numbers up to a thousand, but often have difficulty with larger numbers. It is now common practice to use spaces rather than commas between each group of three figures. eg. 34 000 not 34,000 though the latter will still be found in many text books and should not be considered incorrect.

In reading large figures pupils should know that the final three figures are read as they are written as **hundreds, tens** and **units(ones)**.

Reading from the left, the next three figures are **thousands** and the next group of three are **millions**.

eg. 3 027 251 is three million, twenty seven thousand, two hundred and fifty one.

Order of Operations

It is important that pupils follow the correct order of operations for arithmetic calculations. Most will be familiar with the mnemonic: **BIDMAS**.

Brackets, **I**ndices, **D**ivision, **M**ultiplication, **A**ddition, **S**ubtraction

This shows the order in which calculations should be completed. eg

$$\begin{aligned} 5 + 3 \times 4 \\ \text{means} \\ 5 + 12 \\ = \underline{17} \quad \checkmark \end{aligned}$$

$$\begin{aligned} \textbf{NOT} \quad 5 + 3 \times 4 \\ \text{means } 8 \times 4 \\ = \underline{32} \quad \times \end{aligned}$$

The important facts to remember are that the **B**rackets are done first, then the **I**ndices(powers), **M**ultiplication and **D**ivision and finally, **A**ddition and **S**ubtraction.

eg(i) $(5 + 3) \times 4$
 $= 8 \times 4$
 $= \underline{32}$

eg (ii) $5 + 6^2 \div 3 - 4$
 $= 5 + 36 \div 3 - 4$
 $= 5 + 12 - 4$
 $= 17 - 4$
 $= \underline{13}$

Care must be taken with **S**ubtraction.

$$\begin{aligned} \text{eg } 5 + 12 - 4 \\ = 17 - 4 \\ = \underline{13} \quad \checkmark \end{aligned}$$

$$\begin{aligned} \text{or } 5 + 12 + (-4) \\ = 17 + (-4) \\ = \underline{13} \quad \checkmark \end{aligned}$$

$$\begin{aligned} \text{eg } 5 - 12 + 4 \\ = -7 + 4 \\ = \underline{-3} \quad \checkmark \end{aligned}$$

$$\begin{aligned} \text{but } 5 - 12 + 4 \quad * \\ = 5 - 16 \\ = \underline{-11} \quad \times \end{aligned}$$

* For this to be correct it would have to be written: $5 - (12 + 4)$ so that the bracket is worked out first.

Calculators

Pupils can become over-dependent on the use of calculators for simple calculations, so wherever possible pupils should be encouraged to use mental or pencil and paper methods. It is, however, necessary to give consideration to the ability of the pupil and the objectives of the task in hand. In order to complete a task successfully it may be necessary for pupils to use a calculator for what you perceive to be a relatively simple calculation. This should be allowed if progress within the subject area is to be made. Before completing the calculation pupils should be encouraged to make an estimate of the answer. Having completed the calculation on the calculator they should consider whether the answer is reasonable in the context of the question.

Mental Calculations

Most pupils should be able to carry out the following processes mentally though the speed with which they do it will vary considerably.

- recall addition and subtraction facts up to 20
- recall multiplication and division facts for tables up to 10 x 10.

Pupils should be encouraged to carry out other calculations mentally using a variety of strategies but there will be significant differences in their ability to do so. It is helpful if teachers discuss with pupils how they have made a calculation. Any method which produces the correct answer is acceptable.

$$\text{eg } 53 + 19 = 53 + 20 - 1$$

$$284 - 56 = 284 - 60 + 4$$

$$32 \times 8 = 32 \times 2 \times 2 \times 2$$

$$76 \div 4 = (76 \div 2) \div 2$$

Written Calculations

Pupils often use the '=' sign incorrectly. When doing a series of operations they sometimes write mathematical sentences which are untrue.

eg $5 \times 4 = 20 + 3 = 23 - 8 = 15$ (this is incorrect since $5 \times 4 \neq 15$)

It is important that all teachers encourage pupils to write such calculations correctly.

eg $5 \times 4 = 20$
 $20 + 3 = 23$
 $23 - 8 = \underline{15}$ ✓

The '=' sign should only be used when both sides of an operation have the same value. There is no problem with a calculation such as:

$$43 + 57 = 40 + 3 + 50 + 7 = 90 + 10 = \underline{100}$$
 ✓

since each part of the calculation has the same value.

The '≈' (approximately equal to) sign should be used when estimating answers.

eg $2\,378 - 412 \approx 2\,400 - 400$

$$2\,400 - 400 = \underline{2\,000}$$
 ✓

Pencil & Paper Calculations

All pupils should be able to use some pencil and paper methods involving simple addition, subtraction, multiplication and division. Some less able pupils will find difficulty in recalling multiplication facts to complete successfully such calculations and should refer to the class 'Maths Mat'. In such circumstances it may be more useful to use a calculator in your subject to complete the task.

Before completing any calculation, pupils should be encouraged to estimate a rough value for what they expect the answer to be. This should be done by rounding the numbers and mentally calculating the approximate answer.

After completing the calculation they should be asked to consider whether or not their answer is reasonable in the context of the question.

There is no necessity to use a particular method for any of these calculations and any with which the pupil is familiar and confident should be used.

The following methods are some with which pupils may be familiar.

Addition & Subtraction

Estimate

Addition 3 456 + 975 3 500 + 1 000 = 4 500

$$\begin{array}{r} 3\ 456 \\ + \quad 975 \\ \hline 4\ 431 \\ \hline \end{array}$$

Subtraction by 'counting on'

Estimate

eg 8 003 – 2 569 8 000 – 3 000 = 5 000

Start	Add	
2 569	1	
2 570	30	←
2 600	400	←
3 000	5 000	←
8 000	3	←
8003		←
Total <u>5 434</u>		

Subtraction by decomposition

Estimate

8 000 – 3 000 = 5000

eg $\begin{array}{r} 7\ 9\ 9\ 1 \\ 8\ 0\ 0\ 3 \\ -2\ 5\ 6\ 9 \\ \hline 5\ 4\ 3\ 4 \end{array}$

Addition and subtraction of decimals is completed in the same way but reminders may be needed to maintain place value by keeping decimal points in line underneath each other.

Multiplication and Division by 10,100,1000...

When a number is multiplied by 10 its value has increased tenfold and each digit will move one place to the left so multiplying its value by 10. When multiplying by 100 each digit moves two places to the left, and so on... Any empty columns will be filled with zeros so that place value is maintained when the numbers are written without column headings.

eg. $46 \times 100 = 4\ 600$

T	H	T	U
h			
		4	6
4	6	0	0

The same method is used for decimals.

eg. $5.34 \times 10 = 53.4$

H	T	U	.	t	h
		5	.	3	4
	5	3	.	4	

Empty spaces after the decimal point are not filled with zeros. The place value of the numbers is unaffected by these spaces.

When dividing by 10 each digit is moved one place to the right so making it smaller.

eg. $350 \div 10 = 35$

H	T	U	.	t	h
3	5	0	.		
	3	5	.		

eg. $53 \div 100 = 0.53$

H	T	U	.	t	h
	5	3	.		
		0	.	5	3

When the calculation results in a decimal the units column must be filled with a zero to maintain the place value of the numbers.

Multiplication

$$\begin{array}{r}
 327 \\
 \times 53 \\
 \hline
 981 \leftarrow 327 \times 3 \\
 16350 \leftarrow 327 \times 50 \\
 \hline
 17331
 \end{array}$$

Conventional multiplication as set out above may not suit all pupils and teachers should be aware that other methods may be employed by some pupils.

eg(i) 327×53 Estimate: $300 \times 50 = 15\,000$

X	300	20	7	Total
50	15 000	1000	350	16 350
3	900	60	21	981
Total	15900	1060	371	17331

eg(ii) 456×24 Estimate: $450 \times 20 = 9\,000$

$$\begin{array}{r}
 456 \\
 \times 20 \\
 \hline
 9120 \\
 \text{1 1}
 \end{array}
 +
 \begin{array}{r}
 456 \\
 \times 4 \\
 \hline
 1824 \\
 \text{2 2}
 \end{array}
 =
 \begin{array}{r}
 9120 \\
 +1824 \\
 \hline
 10944
 \end{array}$$

Division

Long

$$\begin{array}{r}
 27 \\
 13 \overline{) 351} \\
 \underline{- 260} \\
 91 \\
 \underline{- 91} \\
 0
 \end{array}$$

Short

$$\begin{array}{r}
 027 \\
 13 \overline{) 351}
 \end{array}$$

Chunking

is a method for Long Division with which some pupils will be familiar and is based on recall of multiplication of numbers by 5, 10, 20 etc. followed by continuous subtraction.

eg $351 \div 13$

$$\begin{array}{r}
 27 \\
 13 \overline{) 351} \\
 \underline{- 130} 10 \\
 221 \\
 \underline{- 130} 10 \\
 91 \\
 \underline{- 52} 4 \\
 39 \\
 \underline{- 39} 3 \\
 027
 \end{array}$$

Any remainders in this type of calculation should be written as a fraction by dividing the remainder by the number by which the calculation has been divided.

Multiplying Decimals

- As always, estimate the answer.
- Complete the calculation as if there were no decimal points.
- In the answer insert a decimal point so that there are the same number of decimal places in the answer as there were in the original question.
- Check to see if the answer is reasonable

eg (i) $1.2 \times 0.3 \approx 1 \times 0.3 = 0.3$

Ignoring the decimal points, this will be calculated as $12 \times 3 = 36$ and will now need two decimal places in the answer.

$$\therefore 1.2 \times 0.3 = 0.36$$

Similarly:

eg (ii) $43.14 \times 3.5 \approx 40 \times 4 = 160$

To work out 43.14×3.5

Multiply 4314 by 35 using any preferred method

$$\begin{array}{r}
 4314 \text{ (43.14 has 2 decimal} \\
 \text{places)} \\
 \times 35 \text{ (3.5 has 1 decimal place)} \\
 \hline
 21570 \\
 129420 + \\
 \hline
 150990
 \end{array}$$

150.990 (3 decimal places need to be put back in the answer)

Answer = 150.99

Percentages

Whilst pupils should be familiar with many operations involving percentages in mathematics lessons it is not proposed to elaborate on all of them. The following is a sample of operations which pupils will be expected to use in other areas.

Calculating percentages of a quantity

Methods for calculating percentages of a quantity vary depending upon the percentage required. Pupils should be aware that fractions, decimals and percentages are different ways of representing part of a whole and know the simple equivalents.

eg $10\% = \frac{1}{10}$ $12\% = 0.12$

Where percentages have simple fraction equivalents, fractions of the amount can be calculated.

- eg. i) To find 50% of an amount, halve the amount.
ii) To find 75% of an amount, find a quarter by dividing by four and then multiply it by three.

Most other percentages can be found by finding 10%, by dividing by 10, and then finding multiples or fractions of that amount

eg. To find 30% of an amount first find 10% by dividing the amount by 10 and then multiply this by three.

$$30\% = 3 \times 10\%$$

Similarly: $5\% = \text{half of } 10\%$ and $15\% = 10\% + 5\%$

Most other percentages can be calculated in this way.

When using the calculator it is usual to think of the percentage as a decimal. Pupils should be encouraged to convert the question to a sentence containing mathematical symbols. ('of means \times)

eg. Find 27% of £350 becomes
 $0.27 \times 350 =$

and this is how it should be entered into the calculator.

Calculating the amount as a percentage

In every case the amount should be expressed as a fraction of the original amount and then converted to a percentage in one of the following ways:

- i) What is 15 as a percentage of 60?
(using simple fractions)

$$\frac{15}{60} = \frac{1}{4} = 25\%$$

- ii) What is 27 out of 50 as a percentage?
(using equivalent fractions)

$$\frac{27}{50} \times 2 = \frac{54}{100} = 54\%$$

- iii) What is 39 as a percentage of 57?
(Using a calculator)

$$\frac{39}{57} = 39 \div 57 = 0.684 \text{ (to 3 d.p.)} = 68.4\%$$

Section 2 – Algebra

The most common use of algebra across the curriculum will be in the use of formulae.

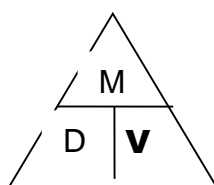
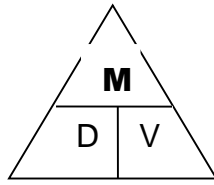
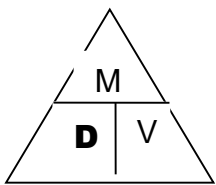
When transforming formulae pupils will be taught to use the ‘balancing’ method where they do the same to both sides of an equation.

eg (i) $A = lb$ Make b the subject of the formula

$$[\div l] \quad \frac{A}{l} = b$$

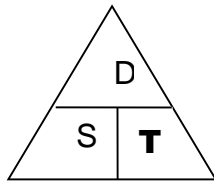
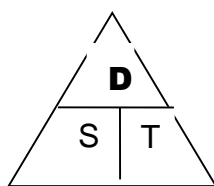
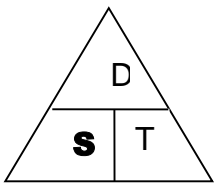
However, triangles can be useful for specific cases **but only** if pupils are struggling to manipulate the formulae, otherwise staff should refer to the balancing method.

eg Density = $\frac{\text{Mass}}{\text{Volume}}$



Density = $\frac{\text{Mass}}{\text{Volume}}$, **Mass** = Density x Volume, **Volume** = $\frac{\text{Mass}}{\text{Density}}$

Similarly with **Distance, Speed and Time**



Speed = $\frac{\text{Distance}}{\text{Time}}$, **Distance** = Speed x Time, **Time** = $\frac{\text{Distance}}{\text{Speed}}$

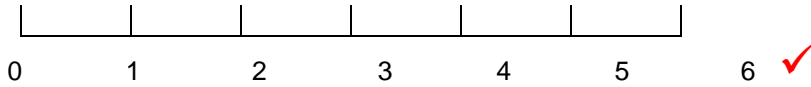
If you want to calculate the speed, cover up S in the triangle and you get $S = \frac{D}{T}$

If you want to calculate the distance, cover up D in the triangle and you get $D = S \times T$

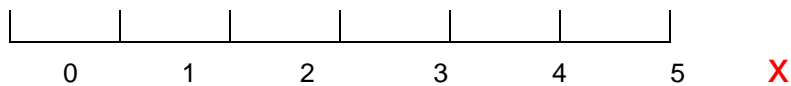
Plotting Points

When drawing a diagram on which points have to be plotted some pupils will need to be reminded that the numbers written on the axes must be on the lines not in the spaces.

eg



NOT



Axes

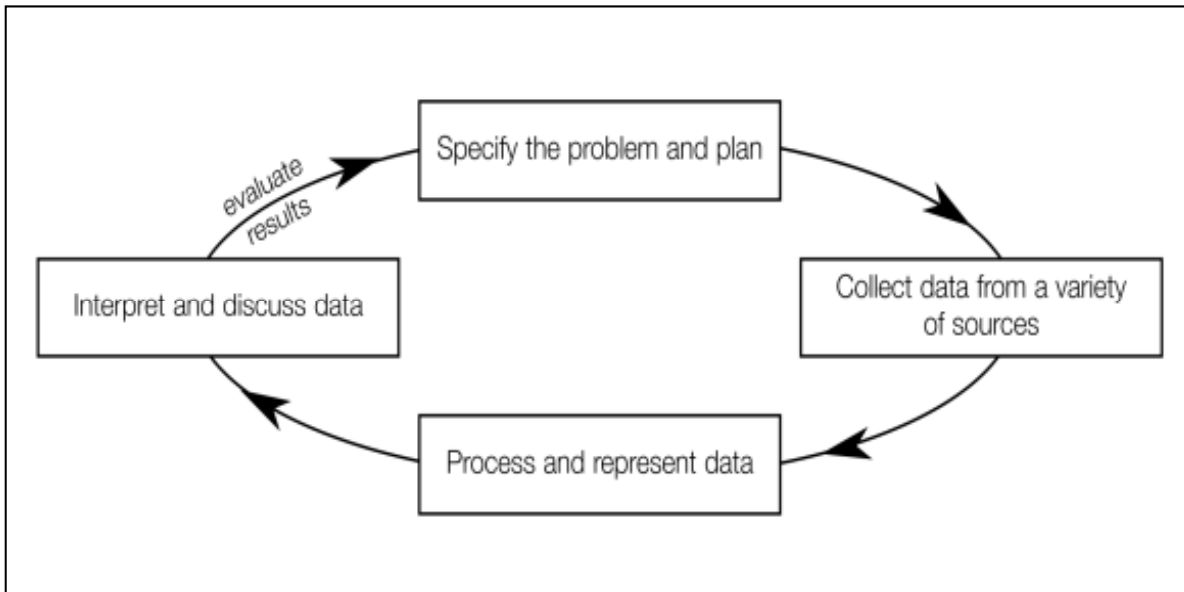
When drawing graphs to represent experimental data it is usual to use the horizontal axis for the variable which has a regular class interval.

eg In an experiment in which temperature is taken every 5 minutes the horizontal axis would be used for time and the vertical axis for temperature.

Having plotted points pupils can sometimes be confused as to whether or not they should join the points. If the results are from an experiment then a 'line of best fit' will usually be needed. Further details appear in the following section on Data Handling.

Section 3 – Data Handling

Pupils use this four stage cycle.



It is important that graphs and diagrams are drawn on the appropriate paper:

- bar charts and line graphs on squared or graph paper.
- pie charts preferably on plain paper.

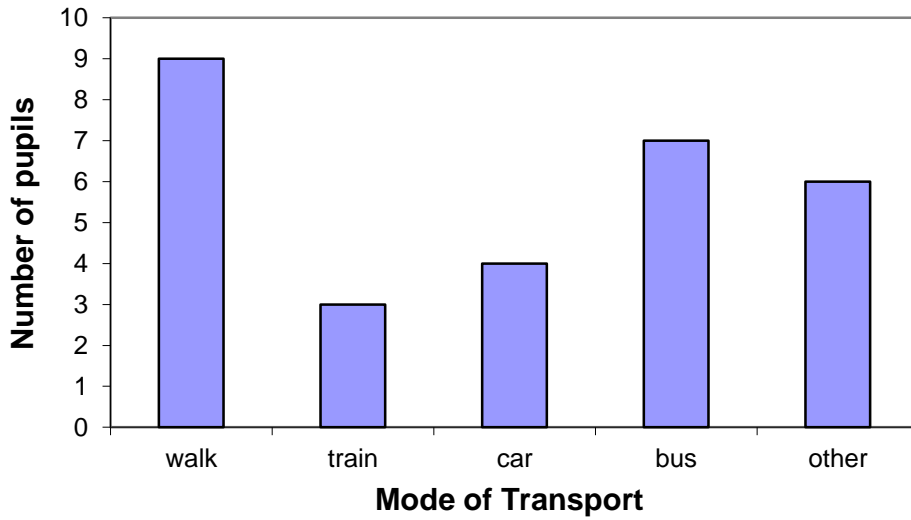
Bar Charts

These are the diagrams most frequently used in areas of the curriculum other than mathematics. The way in which the graph is drawn depends on the type of data to be processed.

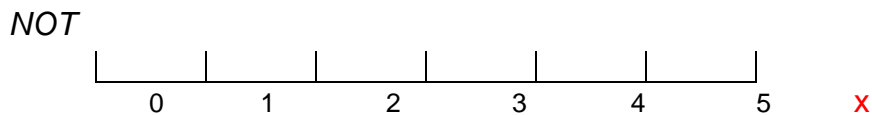
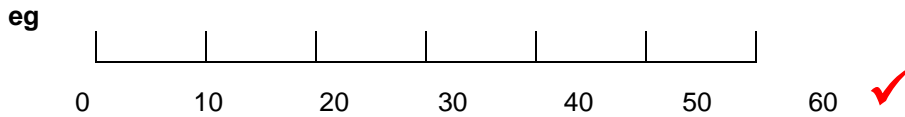
Graphs should be drawn with **gaps between the bars** if the data categories are not numerical (colours, makes of car, names of pop star, etc). There should also be gaps if the data is numeric but can only take a particular value (shoe size, etc). In cases where there are gaps in the graph the horizontal axis will be labelled beneath the columns.

The labels on the vertical axis should be on the lines.
eg.

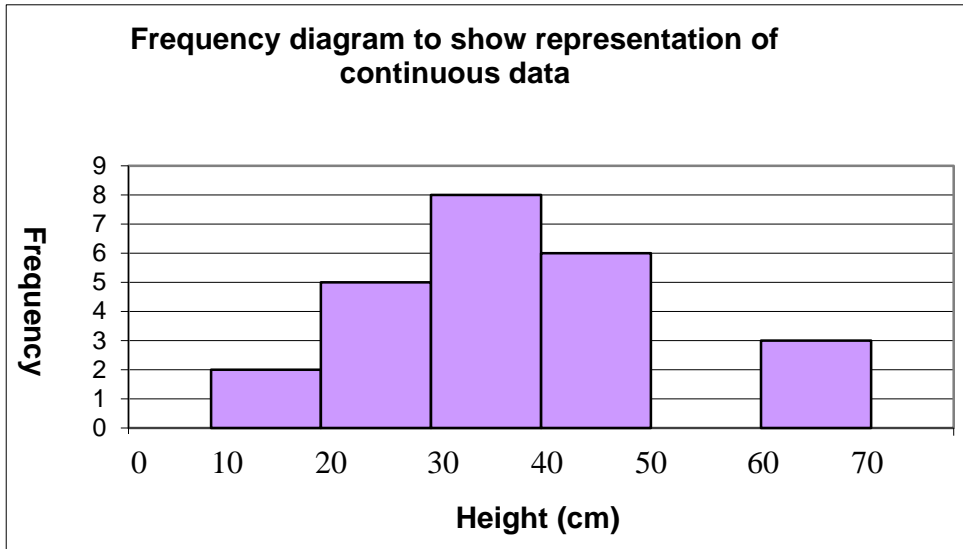
Bar Chart to show representation of non-numerical data



Where the data are continuous, eg. lengths, the horizontal scale should be like the scale used for a graph on which points are plotted.



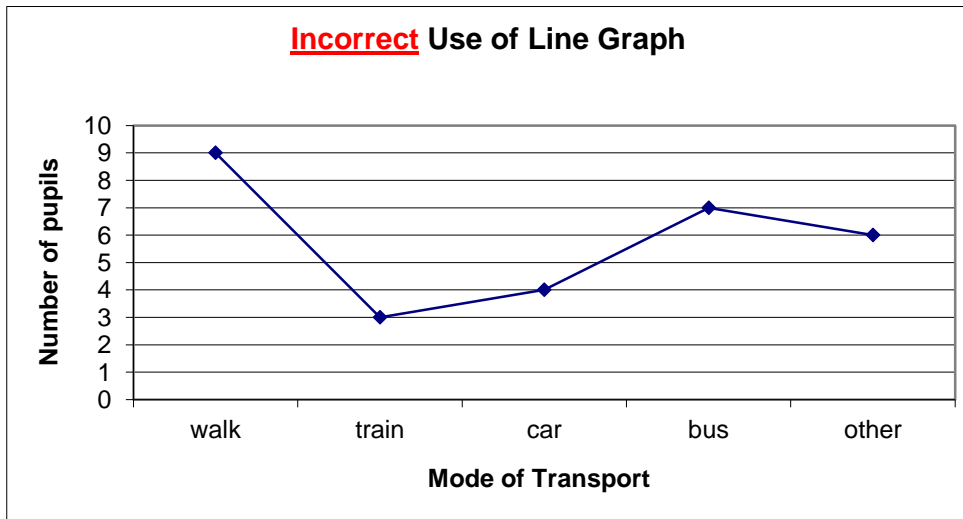
This type of 'bar chart' is a frequency diagram.



Line Graphs

Line graphs should only be used with data in which the order in which the categories are written is significant.

Points are joined if the graph shows a trend or when the data values between the plotted points make sense to be included. For example the measure of a patient's temperature at regular intervals shows a pattern but not a definitive value.



Computer Drawn Graphs & Diagrams

Pupils throughout the school should be able to use **Excel** or other spreadsheets to draw graphs to represent data. Because it is easy to produce a wide variety of graphs there is a tendency to produce diagrams that have little relevance. Pupils should always be encouraged to write a comment explaining their observations from the graph.

Pie Charts

The way in which pupils should be expected to work out angles for a pie chart will depend on the complexity of the question. If the numbers involved are simple it will be possible to calculate simple fractions of 360° .

eg. The following table shows the results of a survey of 30 pupils travelling to school. Show this information on a pie chart.

Mode of Transport	Frequency	Fraction	Angle
Walk	10	$\frac{1}{3}$	120°
Train	3	$\frac{1}{10}$	36°
Car	5	$\frac{1}{6}$	60°
Bus	6	$\frac{1}{5}$	72°
Other	6	$\frac{1}{5}$	72°
Total	30	1	360°

Pie Chart showing pupil modes of transport



However, with more difficult numbers which do not readily convert to a simple fraction pupils should first work out the share of 360° to be allocated to **one** item and then multiply this by its frequency.

eg. 180 pupils were asked their favourite core subject.

Each pupils has $360 \div 180 = 2^\circ$ of the pie chart.

Subject	Number of pupils	Pie Chart Angle
English	63	$63 \times 2 = 126^\circ$
Mathematics	75	$75 \times 2 = 150^\circ$
Science	42	$42 \times 2 = 84^\circ$
Total	180	360°

If the data is in percentage form each item will be represented by 3.6° on the pie. To calculate the angle pupils will need to multiply the frequency by 3.6.

eg. 43% will be represented by $43 \times 3.6 = 154.8^\circ$
 $\approx \underline{155^\circ}$

Any calculations of angles should be rounded to the nearest degree only at the **final stage of the calculation**. If the number of items to be shown is 47 each item will need:

$$360 \div 47 = 7.659574468^\circ$$

This complete number should be used when multiplying by the frequency and then rounded to the nearest degree.

Using Data

Range

The range of a set of data is the difference between the highest and the lowest data values.

eg. If in an examination the highest mark is 80% and the lowest mark is 45%, the range is 35% because $80\% - 45\% = 35\%$

The range is always a **single number**, so it is **NOT** 45% - 80%

Averages

Three different averages are commonly used:

Mean – is calculated by adding up all the values and dividing by the number of values.

Median – is the middle value when a set of values has been arranged in order.

Mode - is the most common value. It is sometimes called the **modal group**.

eg. for the following values: 3, 2, 5, 8, 4, 3, 6, 3, 2,

$$\text{Mean} = \frac{3 + 2 + 5 + 8 + 4 + 3 + 6 + 3 + 2}{9} = \frac{36}{9} = 4$$

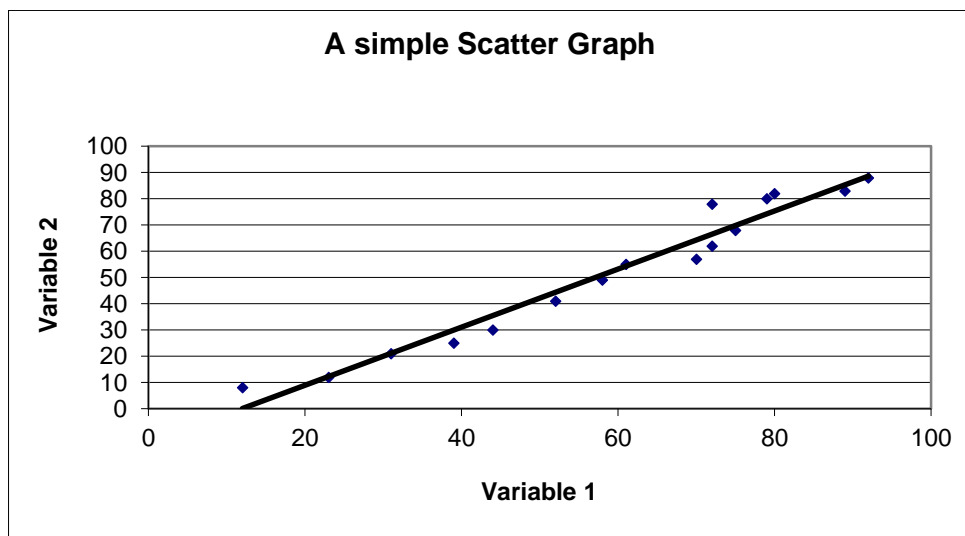
Median – is 3 because 3 is in the middle when the values are put in order.

2, 2, 3, 3, (3), 4, 5, 6, 8

Mode - is 3 because 3 is the value which occurs most often.

Scattergraphs

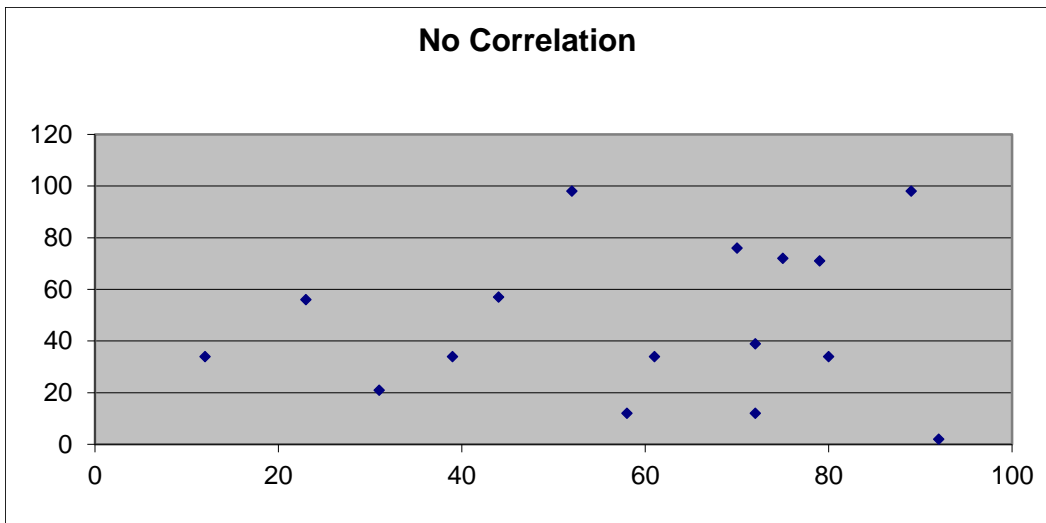
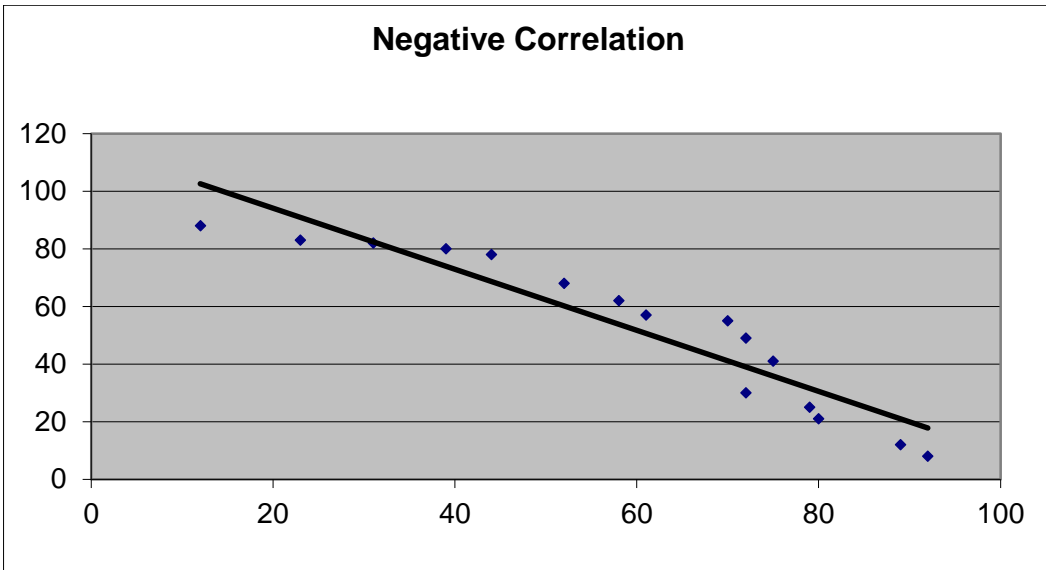
These are used to compare two sets of numerical data. The two values are plotted on two axes labelled as for continuous data. If possible a 'line of best fit' should be drawn.



The degree of correlation between the two sets of data is determined by the proximity of the points to the 'line of best fit'

The above graph shows a positive correlation between the two variables. However you need to ensure that there is a reasonable connection between the two, e.g. ice cream sales and temperature. Plotting use of mobile phones against cost of houses will give two increasing sets of data but are they connected?

Negative correlation depicts one variable increasing as the other decreases; no correlation comes from a random distribution of points. See diagrams below.



Transfer of Skills - Cross Curricular

“It is vital that as the skills are taught, the applications are mentioned and as the applications are taught, the skills are revisited”.

Every effort is made to link Mathematics with other areas of the curriculum. We aim to draw pupils’ attention to these links.

The table below gives some examples.

<p>Art Recognise odd and even numbers The four rules of number Calculate fractions of quantities Simple ratio and proportion Estimating and checking answers Names for 2-D and 3-D shapes/solids Reflective symmetry Metric units of length, capacity, mass and time Drawing 2-D shapes in different orientations Simple scale drawings Recognise 2-D representations of 3-D solids Circumference of a circle Enlargement Spatial awareness</p>	<p>Science %, decimals, fractions Mass/weight Collect, display and interpret data Manipulate and substitute numbers into scientific formulae Speed/distance/time Standard form, large and small numbers Measures Area, Volume Application of the four rules of number Square and square roots Proportional and inverse proportion Lines of best fit (not necessarily straight lines) Ratio</p>	<p>P.E. Estimating measures including reading tape measures (realising ft/in on one side & m/cm on the other) Conversion between metric and Imperial units – particularly ft/in, m/cm and decilitres Reading large numbers Reading time from stop watches Symmetry Graphs Consistency of terms across departments The four rules of number including decimals to 2 d.p. Negative numbers in context Understand angle as a measure of turn Rotational symmetry Interpret simple tables and lists Understand and use the mode, median and the range of a set of data</p>
<p>Music The use of < and > in dynamics The four rules of number Simple ratio and proportion Add and subtract simple fractions (e.g. quavers +semi quavers) Interpreting data (relating to the pop business) Record results in simple lists, tables and block graphs Interpret pie charts</p>	<p>Geography Collect data Display of data Interpretation of data Lines of best fit Rates (e.g. per 1000, per 100 000) Scale Grid references Area/distance Application of the four rules of number</p>	<p>Technology Length (particularly mm) Area Volume Mass/weight Collect, display and interpret data %, decimals and fractions Costings Application of the four rules of number</p>
<p>ICT/Business Studies Use of spreadsheets Formulae Use of money Profit and loss, break even %, decimals and fractions Display and interpretation of data Application of the four rules of number</p>	<p>History/RE Islamic patterns Subtraction by decomposition for dates (incl.BC and AD) Large numbers – meaning of billion Centuries (e.g. 845 is in the 9th Century)</p>	<p>Modern Foreign Languages Measures of length/distance, time and weight, counting, tables, exchange rates, Money/costs. Speeds, distance.</p>

